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Effect of velocity-dependent nuclear forces on (γ, p) disintegration of the alpha particle

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Abstract. We calculate the total cross section $\sigma(\gamma, \mathrm{p})$ for the two-particle photodisintegration of the alpha particle. We describe the ground states of the alpha particle and the triton by modified Irving wave functions, the parameters of which have been determined from variational calculations of the binding energies of these systems, using a central velocity-dependent potential. We neglect the final-state interaction. We compare our results with the recent experiments of Gorbunov and with other similar calculations of Gunn and Irving and Bransden *et al.* We find that, though the velocity-dependent forces cause the total cross section $\sigma(\gamma, \mathrm{p})$ to change in the right direction, the discrepancy of fitting simultaneously the binding energy, r.m.s. radius and the maximum cross section at the correct energy is not removed altogether.

1. Introduction

The study of photodisintegration of the lightest nuclei, in principle, yields valuable information regarding the nuclear wave functions and interactions. Recently the authors (Srivastava and Jain 1967) have evaluated the two moments for the photoeffect, i.e. the integrated cross section ($\sigma_{\mathrm{int}} = \int_0^\infty \sigma(W)\,\mathrm{d}W$) and bremsstrahlung-weighted cross section ($\sigma_{\mathrm{b}} = \int_0^\infty (\sigma/W)\,\mathrm{d}W$) for the alpha particle (^{4}He), by using the sum rules of Levinger and Bethe (1950). In this calculation we have described the ground state of ^{4}He by a four-parameter modified Irving wave function, the parameters of which are obtained from a variational calculation of the binding energy of ^{4}He using a central velocity-dependent potential. Although our results for σ_{int} and σ_{b} are in reasonable agreement with the experimental values of Gorbunov and Spiridonov (1958), this calculation does not give a detailed idea of the energy dependence of the total cross section and its maximum value above the threshold. In the present paper we wish to investigate these aspects of the photoeffect of ^{4}He.

One of the most suitable reactions for investigation is the two-particle (γ, p) photodisintegration of ^{4}He, as the cross section for this reaction has been measured in detail by Gorbunov (1967, private communication, see also Gorbunov 1967). The theory of this reaction has been developed by Gunn and Irving (1951). They have used wave functions of the following forms:

Gaussian

$$\psi = N \exp\left\{-\mu^2 \left(\sum_{i<j} r_{ij}^2\right)\right\} \tag{1}$$

Gunn–Irving

$$\psi = \frac{N \exp\{-\mu(\Sigma_{i<j} r_{ij}^2)^{1/2}\}}{(\Sigma_{i<j} r_{ij}^2)^{1/2}} \tag{2}$$

to describe the ground states of ^{4}He and ^{3}H and have assumed the ejected proton to be a plane wave. They find that when they determine the parameters of these wave functions from respective variational calculations of these systems, with a purely central potential which fits the low-energy nuclear data and the binding energy of the deuteron, the calculated cross section exhibits a low maximum (Gunn and Irving 1951) at an energy much higher than that observed (Bransden *et al.* 1957). To obtain the maximum cross section at the correct energy, the size of the ^{4}He nucleus has to be increased considerably (i.e. the parameter of the ^{4}He wave function is decreased) but when this is done the calculated cross section becomes several times larger.

In order to remove this discrepancy Bransden *et al.* (1957) have recalculated the (γ, p) cross section by using the Irving wave function of the form

$$\psi = N \exp\left\{-\mu\left(\sum_{i<j} r_{ij}^2\right)^{1/2}\right\} \tag{3}$$

and a mixture of central and tensor forces. However, this calculation also leads to the conclusion that it is impossible to choose the parameters of these types of wave function (i.e. Gaussian, Irving, etc.) in such a way as to give simultaneously the correct values of the binding energy, r.m.s. radius and the correct value of the maximum cross section at the correct energy. They further remark that this discrepancy may be removed by the inclusion of hard-core forces.

During the past few years a number of workers (Razavy *et al.* 1962, Rojo and Simmons 1962, Green 1962) have shown that velocity-dependent potentials give as good a fit to the relevant two-body data as the hard-core potentials. Therefore, in view of the remark of Bransden *et al.* (1957), it is of interest to calculate the (γ, p) cross section of ^{4}He by using a velocity-dependent nuclear potential.

2. Calculation of the total cross section $\sigma(\gamma, p)$ for ^{4}He

We consider the two-particle photodisintegration of the alpha particle given by the reaction

$$\gamma + {}^4\mathrm{He} \rightarrow {}^3\mathrm{H} + \mathrm{p}. \tag{4}$$

We neglect the final-state interaction and represent the ejected proton in the final state by a plane wave; the modified Irving wave functions ψ_α and ψ_T describe respectively the alpha particle in the initial state and triton in the final state:

$$\psi_\alpha = N_\alpha\left[\frac{\exp\{-\mu_\alpha(\Sigma_{i<j} r_{ij}^2)^{1/2}\} + A_\alpha \exp\{-\lambda_\alpha(\Sigma_{i<j} r_{ij}^2)^{1/2}\}}{(\Sigma_{i<j} r_{ij}^2)^{n_\alpha}}\right], \quad i,j = 1, 2, 3 \text{ and } 4 \tag{5}$$

$$\psi_\mathrm{T} = N_\mathrm{T}\left[\frac{\exp\{-\mu_\mathrm{T}(\Sigma_{i<j} r_{ij}^2)^{1/2}\} + A_\mathrm{T} \exp\{-\lambda_\mathrm{T}(\Sigma_{i<j} r_{ij}^2)^{1/2}\}}{(\Sigma_{i<j} r_{ij}^2)^{n_\mathrm{T}}}\right], \quad i,j = 2, 3 \text{ and } 4. \tag{6}$$

The parameters of these wave functions have been determined from a variational calculation of the binding energies of the alpha particle and the triton with a central velocity-dependent potential (Srivastava 1965) given by

$$\begin{aligned} V_{\mathrm{eff}}(r) = & -\tfrac{1}{2}(1+X_{\mathrm{stat}})(V_0)_{\mathrm{stat}} \exp\left(\frac{-2r}{\beta_\mathrm{s}}\right) \\ & + \frac{(V_0)_{\mathrm{vel}}}{2M}\left\{p^2 \exp\left(\frac{-2r}{\beta'_\mathrm{s}}\right) + \exp\left(\frac{-2r}{\beta'_\mathrm{s}}\right) p^2\right\} \\ & + \frac{X_{\mathrm{vel}}(V_0)_{\mathrm{vel}}}{2M}\left\{p^2 \exp\left(\frac{-2r}{\beta'_\mathrm{t}}\right) + \exp\left(\frac{-2r}{\beta'_\mathrm{t}}\right) p^2\right\} \end{aligned} \tag{7}$$

where the values of the potential parameters, consistent with the p–p low- and high-energy scattering data, binding energy of the deuteron and Breit's ^{3}S phase shifts at $E_{\mathrm{lab}} = 147$, 270 and 310 MeV, are (Srivastava 1965)

$$\begin{aligned} &(V_0)_{\mathrm{stat}} = 100\,\mathrm{MeV}, && (V_0)_{\mathrm{vel}} = 2 \\ &X_{\mathrm{stat}} = 1{\cdot}84, && X_{\mathrm{vel}} = 0{\cdot}55 \\ &\frac{1}{\beta_\mathrm{s}} = 0{\cdot}625\,\mathrm{fm}^{-1}, && \frac{1}{\beta'_\mathrm{s}} = 1{\cdot}4\,\mathrm{fm}^{-1}, \quad \frac{1}{\beta'_\mathrm{t}} = 1\,\mathrm{fm}^{-1}. \end{aligned} \tag{8}$$

The best values of the parameters in wave functions (5) and (6) obtained in the above variational calculations are (Srivastava and Jain 1967, Jain and Srivastava 1968 and 1968, unpublished)

$$n_\alpha = 0, \quad \mu_\alpha = 0{\cdot}90\,\mathrm{fm}^{-1}, \quad \lambda_\alpha = 1{\cdot}14\,\mathrm{fm}^{-1} \text{ and } A_\alpha = -1{\cdot}38 \tag{9}$$

$$n_\mathrm{T} = 0, \quad \mu_\mathrm{T} = 0{\cdot}70\,\mathrm{fm}^{-1}, \quad \lambda_\mathrm{T} = 1{\cdot}23\,\mathrm{fm}^{-1} \text{ and } A_\mathrm{T} = -1{\cdot}20. \tag{10}$$

The corresponding values of the binding energy and r.m.s. radius are shown in table 1.

Table 1. Binding energy and r.m.s. radius for ^{4}He and ^{3}H

		Binding energy (MeV)	r.m.s. radius (fm)
^{4}He	calculated	30·1	1·55
	experimental	28·3	1·44
^{3}H	calculated	8·22	1·68
	experimental	8·49	1·64

We consider only the electric-dipole transitions in the reaction given by equation (4). Then the transition probability per unit time for disintegration into a group of final states F by a quantum energy $h\nu$, with its polarization vector along the z axis is given by (Gunn and Irving 1951)

$$\omega_{\mathrm{fi}} = \pi^2 ke^2 \left| \int \psi_\mathrm{T}(\boldsymbol{v}, \boldsymbol{w}) u_z \exp(-\mathrm{i}\boldsymbol{p}.\boldsymbol{u}) \psi_\alpha(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w})\, \mathrm{d}\boldsymbol{u}\, \mathrm{d}\boldsymbol{v}\, \mathrm{d}\boldsymbol{w} \right|^2 \rho_\mathrm{E}(F) \tag{11}$$

where we have used the transformations

$$\boldsymbol{u} = -\boldsymbol{r}_1 + \tfrac{1}{3}(\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4), \qquad \boldsymbol{v} = \tfrac{1}{2}(\boldsymbol{r}_3+\boldsymbol{r}_4) - \tfrac{1}{3}(\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4)$$

and

$$\boldsymbol{w} = \boldsymbol{r}_3 - \boldsymbol{r}_4. \tag{12}$$

Finally we convert the transition probability into total cross section $\sigma(\gamma, \mathrm{p})$ and, using the wave functions given by equations (5) and (6), we obtain

$$\begin{aligned}\sigma(\gamma, \mathrm{p}) = {} & 11^2 \times 9^2 \times 7^2 \times 5\sqrt{3}\pi e^2 k p^3 M \\ & \times [\mu_\alpha^{-9} + 2A_\alpha\{\tfrac{1}{2}(\mu_\alpha+\lambda_\alpha)\}^{-9} + A_\alpha^2 \lambda_\alpha^{-9}]^{-1} \\ & \times [\mu_\mathrm{T}^{-6} + 2A_\mathrm{T}\{\tfrac{1}{2}(\mu_\mathrm{T}+\lambda_\mathrm{T})\}^{-6} + A_\mathrm{T}^2 \lambda_\mathrm{T}^{-6}]^{-1} \\ & \times [F(\mu_\mathrm{T}, \mu_\alpha) + A_\mathrm{T}F(\lambda_\mathrm{T}, \mu_\alpha) + A_\alpha F(\mu_\mathrm{T}, \lambda_\alpha) + A_\mathrm{T}A_\alpha F(\lambda_\mathrm{T}, \lambda_\alpha)]^2. \end{aligned} \tag{13}$$

$F(T, H)$ in equation (13) is given as follows:

(i) If $\tfrac{1}{3}p^2 + H^2 > \tfrac{3}{4}T^2$ and $z = (\tfrac{1}{3}p^2+H^2)/(\tfrac{1}{3}p^2+H^2-\tfrac{3}{4}T^2)$ then

$$F(T, H) = \frac{TH}{(\tfrac{1}{3}p^2+H^2-\tfrac{3}{4}T^2)^{13/2}} \left\{ \frac{1}{24(z-1)^{1/2}} (585z^2 - 975z + 422) - \frac{1}{11 \times 9 \times 7} \frac{1}{z^3(z-1)^{1/2}} (594z^2 + 66z + 8) - \frac{5}{8}(39z^2 - 52z + 16) \sin^{-1}\left(\frac{1}{\sqrt{z}}\right) \right\}.$$

(ii) If $\tfrac{1}{3}p^2 + H^2 = \tfrac{3}{4}T^2$ then

$$F(T, H) = \frac{TH}{17 \times 13 \times 11 \times 63 \times 32} \left(\frac{16}{3T^2}\right)^{13/2}.$$

(iii) If $\frac{1}{3}p^2+H^2 < \frac{3}{4}T^2$ and $z = (\frac{1}{3}p^2+H^2)/(\frac{3}{4}T^2-\frac{1}{3}p^2-H^2)$ then

$$F(T,H) = \frac{TH}{(\frac{3}{4}T^2-\frac{1}{3}p^2-H^2)^{13/2}}\left[\frac{1}{24(z+1)^{1/2}}(585z^2+975z+422) + \frac{1}{11\times 9\times 7}\,\frac{1}{z^3(z+1)^{1/2}}(594z^2-66z+8) - \frac{5}{16}(39z^2+52z+16)\ln\left\{\frac{(z+1)^{1/2}+1}{(z+1)^{1/2}-1}\right\}\right].$$

Using the parameters given by equations (9)–(10), we have made numerical estimates for the total cross section $\sigma(\gamma, \mathrm{p})$ (cf. equation (13)) between the energies 19·8 MeV (threshold energy) and 170 MeV.

3. Discussion of the results and conclusion

Figure 1 shows our calculated cross section along with the recent experimental results of Gorbunov (1967, private communication, see also Gorbunov 1967) and the theoretical results of Gunn and Irving (1951) and Bransden *et al.* (1957). Our calculation (curve A) gives the maximum cross section of 2·2 mbn at an energy of 15·2 MeV above the threshold, while the experimental cross section $1{\cdot}94 \pm 0{\cdot}14$ mbn appears in the energy range 4·2–5·2 MeV above threshold.

Curve B has been obtained by Gunn and Irving (1951) using the wave function given by equation (2) to describe the triton and the alpha particle, respectively. (The parameter μ in equation (2) is denoted by μ_T for the triton and μ_α for the alpha particle.) For curve B Gunn and Irving (1951) have chosen $1/\mu_T = 2{\cdot}5$ fm and $1/\mu_\alpha = 1{\cdot}7$ fm to give the correct values of the Coulomb energy of ^{3}He and to fit the binding energy of ^{4}He, respectively. In their calculation Gunn and Irving (1951) have neglected the final-state interaction.

Curve C shows the results of Bransden *et al.* (1957). They have used the same variational procedure as we have. Their trial wave function is of the form given by equation (3) with μ as variational parameter. But, unlike us, they have taken tensor forces into consideration. Their variational calculation gives $\mu_\alpha = 1{\cdot}134\ \mathrm{fm}^{-1}$ and the corresponding value of the binding energy is 16·6 MeV. They find that the $(\sigma, h\nu)$ curve is insensitive to μ_T and, therefore, they have taken $\mu_T = \mu_\alpha = 1{\cdot}134\ \mathrm{fm}^{-1}$ in curve C.

According to Bransden *et al.* (1957) the interaction in the final state is not responsible for the discrepancy between the wave functions that are consistent with the binding energy of ^{4}He and those that are consistent with the (γ, p) cross section. In view of this remark of Bransden *et al.* (1957), we have neglected final-state interaction in our calculation. Since the Coulomb interaction between the outgoing proton and the triton is negligible for quanta above 22 MeV and the threshold energy is about 20 MeV, therefore, we have also neglected this interaction in our calculation.

Table 1 shows that our variational calculation gives results for the binding energy and r.m.s radius in agreement with experiments both for ^{4}He and ^{3}H. In connection with earlier calculations of the binding energy it has been noticed by Rustgi and Levinger (1957) and independently by Dalitz and Ravenhall (Hofstadter 1956) that, though the agreement with the binding energy was good, the r.m.s. radius was only $\frac{2}{3}$ of the experimental value. Both Gunn and Irving (1951) and Bransden *et al.* (1957) are unable to fit simultaneously the r.m.s. radius and the binding energy of the alpha particle. Even for more elaborate calculations of Clark (1954) this discrepancy is not removed. It is satisfactory to note that we obtain reasonably good values both for the binding energy and the r.m.s. radius of the alpha particle.

As remarked earlier, Gunn and Irving (1951) obtain very small values of the $\sigma(\gamma, \mathrm{p})$ cross section when they use the parameters obtained from the variational calculation of ^{4}He and ^{3}H.

A comparison of our results for $\sigma(\gamma, \mathrm{p})$ with those of Bransden *et al.* (1957) shows that by using velocity-dependent nuclear forces we are able to fit simultaneously the binding energy, r.m.s. radius and the maximum cross section at the correct energy for the alpha particle much better than Bransden *et al.* (1957). However, in our case also, the discrepancy of fitting simultaneously the binding energy, r.m.s. radius and the maximum cross section at the correct energy is not removed altogether. One may expect better results by considering a tensor velocity-dependent potential which will lower the binding energy of ^{4}He and

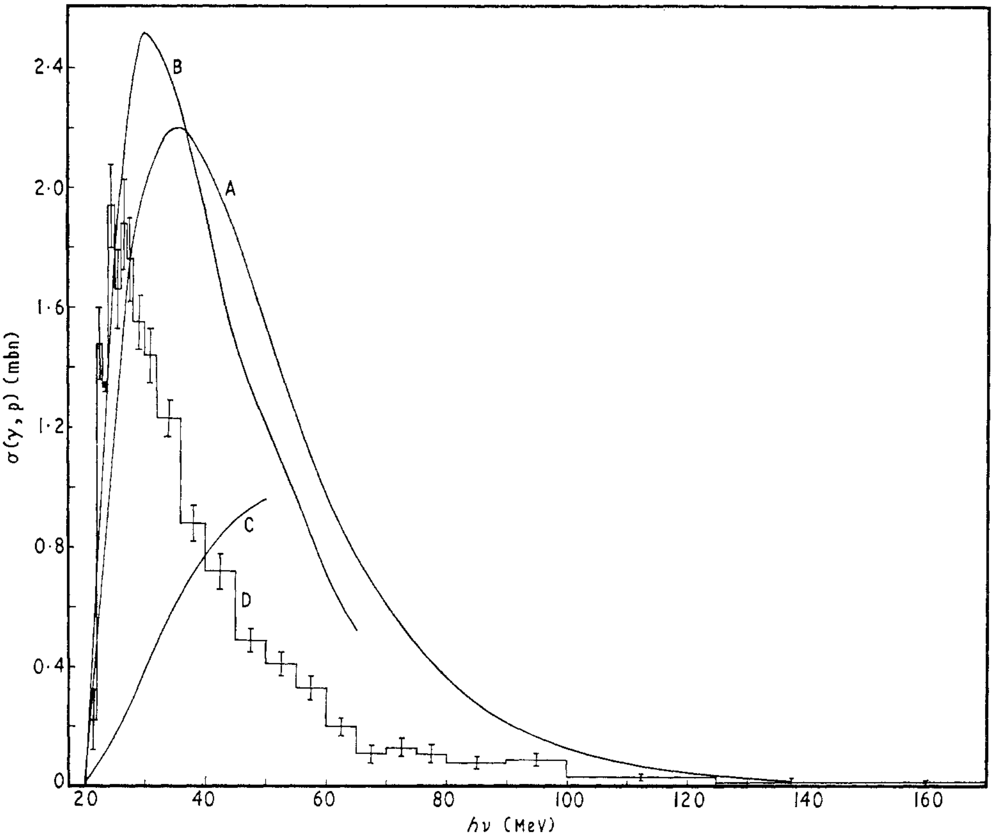

Figure 1. Cross sections for the two-particle (γ, p) disintegration of ^{4}He. Curve A, present calculation using a central velocity-dependent potential and modified Irving wave functions both for ^{4}He and ^{3}H, whose parameters are obtained from a variational calculation. These give results for the binding energy and r.m.s. radii of the two systems in agreement with experiments. Curve B, Gunn and Irving (1951). Their calculation uses purely central potential and Gunn–Irving wave functions whose parameters fit the binding energy of ^{4}He and Coulomb energy of ^{3}He. Curve C, Bransden *et al.* (1957). Their calculation uses a mixture of central and tensor potential and the Irving wave function whose parameter is consistent with the binding energy of ^{4}He, obtained from a variational calculation. Further, they use $\mu_T = \mu_\alpha$. Curve D, histogram corresponding to the experimental results of Gorbunov (1967, private communication, see also Gorbunov 1967).

correspondingly increase its r.m.s. radius. This will cause the maximum cross section to shift in the right direction, i.e. the maximum cross section will now appear at lower energy. But the maximum value of the $\sigma(\gamma$, p) cross section is likely to be greater than the experimental value.

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